SYLLABUS UNDER AUTONOMY

MATHEMATICS

SEMESTER III **COURSE: A.MAT.3.01**

Calculus and Analysis [45 LECTURES]

LEARNING OBJECTIVES :

To learn about i) lub axiom of R and its consequences
ii) Convergence of sequences
iii) Convergence of infinite series

Unit 1. Real Numbers (15 Lectures)

(a) Statements of algebraic and order properties of R.
(i) Elementary consequences of these properties including the A.M. - G.M. inequality, Cauchy-Schwarz inequality and Bernoulli inequality (without proof).

(b) (i) Review of absolute value and neighbourhood of a real number.
(ii) Hausdorff property.

(c) Supremum (lub) and infimum (glb) of a subset of R, lub axiom of R.
Consequences of lub axiom of R including
(i) Archimedian property.
(ii) Density of rational numbers.
(iii) Existence of nth root of a positive real number (in particular square root).
(iv) Decimal representation of a real number.

(d) (i) Nested Interval Theorem.
(ii) Open sets in R and closed sets as complements of open sets.
(iii) Limit points of a susbset of R, examples, characterisation of a closed set as a set containing all its limit points.

(e) Open cover of a susbset of R, Compact susbset of R, Definition and examples.
A closed and bounded interval [a, b] is compact.

Reference for Unit 1: Chapter II, Sections 1, 2, 4, 5, 6 and Chapter X, Sections 1, 2 of Introduction to Real Analysis, Robert G. Bartle and Donald R. Sherbet, Springer Verlag.

Unit 2. Sequences, Limits and Continuity (15 Lectures)

(a) Sequence of real numbers, Definition and examples. Sum, difference, product, quotient and scalar multiple of sequences.

(b) Limit of a sequence, Convergent and divergent sequences, Uniqueness of limit of a convergent sequence, Algebra of convergent sequences, Sandwich theorem of sequences. Limits of standard sequences such as

$\left\{\frac{1}{n^{\alpha}}\right\}\alpha > 0$, {an } |a| < 1, {n1/n }, {a1/n } a > 0, {1/ n!},{an/n!} where $a \in R$

Examples of divergent sequences.

(c) (i) Bounded sequences. A convergent sequence is bounded.

(ii) Monotone sequences, Convergence of bounded monotone sequences. The number e as a limit of a sequence, Calculation of square root of a positive real number.

(d) (i) Subsequences.

(ii) Limit inferior and limit superior of a sequence.

(iii) Bolzano-Weierstrass theorem of sequences.

(iv) Sequential characterisation of limit points of a set.

(e) Cauchy sequences, Cauchy completeness of R.

(f) Limit of a real valued function at a point

(i) Review of the $\varepsilon - \delta$ definition of limit of functions at a point, uniqueness of limits of a function at a point whenever it exists.

(ii) Sequential characterization for limits of functions at a point, Theorems of limits (Limits of sum, difference, product, quotient, scalar multiple and sandwich theorem).

(iii) Continuity of function at a point, $\varepsilon-\delta$ definition, sequential criterion, Theorems about continuity of sum , difference , product , quotient and scalar multiple of functions at a point in the domain using $\varepsilon - \delta$ definition or sequential criterion. Continuity of composite functions. Examples of limits and continuity of a function at a point using sequential criterion.

(iv) A continuous function on closed and bounded interval is bounded and attains bounds.

Reference for Unit 2 : Chapter III, Sections 1, 2, 3, 4, 5, Chapter IV, Sections 1, 2 & Chapter V, Sections 1, 2, 3 of Introduction to Real Analysis, Robert G. Bartle and Donald R. Sherbet, Springer Verlag.

Unit 3. Infinite Series (15 Lectures)

(a) Infinite series of real numbers, the sequence of partial sums of an infinite series, convergence and divergence of series, sum, difference and multiple of convergent series are again convergent.
(b) Cauchy criterion of convergence of series. Absolute convergence of a series, Geometric series.
(c) Alternating series, Leibnitz' Theorem, Conditional convergence. An absolutely convergent series is conditionally convergent, but the converse is not true.
(d) Rearrangement of series (without proof). Cauchy condensation test (statement only),application to convergence of p- series ($p > 1$). Divergence of Harmonic series
(e) Tests for absolute convergence, Comparison test, Ratio test, Root test.
(f) Power series, Radius of convergence of power series:- The exponential, sine and cosine series.
(g) Fourier series, Computing Fourier Coefficients of simple functions such as x, x2 , |x|, piecewise continuous functions on $[-\pi, \pi]$.

Reference for Unit 3: Chapter IX, Sections 1, 2, 3, 4 and Chapter VIII, Sections 3, 4 of Introduction to Real Analysis, Robert G. Bartle and Donald R. Sherbet, Springer Verlag.

Recommended Books

1. Robert G. Bartle and Donald R. Sherbet : Introduction to Real Analysis, Springer Verlag.
2. R. Courant and F. John : Introduction to Calculus and Analysis Vol I, Reprint of First Edition, Springer Verlag, New York 1999.
3. R. R. Goldberg: Methods of Real Analysis, Oxford and IBH Publication Company, New Delhi.
4. T. Apostol: Calculus Vol I, Second Edition, John Wiley.
5. M. H. Protter: Basic elements of Real Analysis, Springer Verlag, New York 1998.

Additional Reference Books

1. Howard Anton, Calculus - A new Horizon, Sixth Edition, John Wiley and Sons Inc,1999.
2. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company, 1994.

Suggested topics for Tutorials/Assignments

(1) Properties of real numbers and Hausdorff property.
(2) Bounded sets, finding l.u.b. and g.l.b. of sets.
(3) Archmedian Property and Density Theorem.
(4) Finding limit points of given sets
(5) Compact sets.

(6) (i) Find limits of sequences using definition.
(ii) Monotone sequences.
(7) Subsequences, finding limit inferior and limit superior of given sequences.
(8) Cauchy sequences.
(9) Limits and continuity using sequential criterion.
(10) Convergence of series. Comparison test.
(11) Convergence of series: Root test, Ratio test.
(12) Alternating series, tests for absolute convergence
(13) Radius of convergence of a power series.
(14)Fourier Series.

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SYLLABUS UNDER AUTONOMY

MATHEMATICS

SEMESTER 3 **Course:A.MAT.3.02**

Linear Algebra

Learning Objectives: To solve system of equations using row echelon form of a matrix, To study the structure of a vector space through its basis and to understand Gram-Schmidth orthogonalisation process in an inner product space.

Unit 1. System of linear equations and matrices

(a) System of homogeneous and non-homogeneous linear equations.
The solution of system of m homogeneous equations in n unknowns by elimination and their geometric interpretation for (m,n) = (1,2),(1,3),(2,2),(2,3),(3,3).
Definition of n tuples of real nos., sum of two n tuples and scalar multiple of n tuple.
The existence of non-trivial solution of such a system for $m<n$. The sum of two solutions and a scalar multiple of a solution of such a system is again a solution of the system.

(b) Matrices over R, the matrix representation of system of homogeneous and non homogeneous linear equations.
Addition, scalar multiplication and multiplication of matrices, transpose of a matrix.
Types of matrices. Transpose of product of matrices, invertible matrices, product of invertible matrices.

(c) Elementary row operations on matrices, row echelon form of a matrix, Guassian elimination method. Application of Guass elimination method to solve system of linear equations.
Row operations and elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

Unit 2. Vector spaces over R.

(a) Definition of vectot space over R. Eg such as: Euclidean space R^n, space of R^∞ of sequences over R, space of $m \times n$ matrices over R, the space of polynomials with real co-efficients, space of real valued functions on a nonempty set.

(b) Subspaces- definition and examples including: lines in R^2, lines and planes in R^3, solutions of homogeneous system of linear equations, hyperplane, space of convergent sequences, space of symmetric and skew symmetric, upper triangular, lower triangular, diagonal matrices, and so on.

(c) Sum and intersection of subspaces, direct sum of vector spaces, linear combination of vectors, convex sets, linear span of a subset of a vector space, linear dependence and independence of a set.

(d) (For finitely generated vectot spaces only) Basis of a vector space , basis as a maximal linearly independent set and as a minimal set of generators, dimension of a vector space.

(e) Row space, column space of an $m \times n$ matrix over and row rank, column rank of a matrix. Equivalence of row rank and column rank, computing rank of a matrix by row reduction.

Unit 3. Inner product spaces.

(a) Dot product in R^n, defn of general inner prduct on a vector space over R. Eg such as C[-∏,∏] and so on.

(b) Norm of a vector in an inner product space, Cauchy-schwartz inequality, triangle inequality.
Orthogonality of vectors, Pythagoras theorem and geometric application in $R^{2,}$ projection of a line, projection being the closest approximation.
Orthogonal complements of a subspace, orthogonal complement in R^2 and R^3.
Orthogonal sets and orthonormal sets in an inner product space.Orthogonal and orthonormal bases. Gram-Schmidth orthogonalisation process, egs in R^2 , R^3 and R^4.

References:1. Introduction to linear algebra by Serge Lang, Springer verlag. 2. Linear algebra a geometric approach by Kumaresan, Prentice-hall of India private limited, New Delhi. 3. Linear algebra by Hoffman and Kunze, Tata McGraw-Hill, New Delhi.