

## SYLLABUS UNDER AUTONOMY

### MATHEMATICS

**SEMESTER IV**

**COURSE: A.MAT.4.01**

**Calculus and Analysis**

**[45 LECTURES]**

Reference for Unit 1: Chapter 2, Sections 7, 8, 9, 10 and Chapter 3, Sections 14, 15, 16, 17, 18, 19, 20 of Differential Equations with Applications and Historical Notes, G.F. Simmons, McGraw Hill.

Unit 2. Multiple integrals (15 Lectures)

Review of functions of two and three variables, partial derivatives and gradient of two or three variables.

(a) Double integrals:

(i) Definition of double integrals over rectangles.

(ii) Properties of double integrals.

(iii) Double integrals over bounded regions.

(b) Statement of Fubini's Theorem, Double integrals as volumes.

(c) Applications of Double integrals: Average value, Areas, Moments, Center of Mass.

(d) Double integrals in polar form.

(e) Triple integrals in Rectangular coordinates, Average, volumes.

(f) Applications of Triple integrals: Mass, Moments, Parallel axis Theorem.

(g) Triple integrals in Spherical and Cylindrical coordinates.

Reference for Unit 2: Chapter 13, Sections 13.1, 13.2, 13.3, 13.4, 13.5, 13.6 of Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, Ninth Edition, Addison-Wesley, 1998.

Unit 3. Integration of Vector Fields (15 Lectures)

(a) Line Integrals, Definition, Evaluation for smooth curves.

Mass and moments for coils, springs, thin rods.

(b) Vector fields, Gradient fields, Work done by a force over a curve in space, Evaluation of work integrals.

(c) Flow integrals and circulation around a curve.

(d) Flux across a plane curve.

(e) Path independence of the line integral of  $F$  region,  $F$  being a vector field  
Conservative fields, potential function.

(f) The Fundamental theorems of line integrals (without proof).

(g) Flux density (divergence), Circulation density (curl) at a point.

(h) Green's Theorem in plane (without proof), Evaluation of line integrals using Green's Theorem.

Reference for Unit 3: Chapter 14 of 14.1, 14.2, 14.3, 14.4 Calculus and Analytic Geometry, G.B. Thomas and R. L. Finney, Ninth Edition, Addison-Wesley, 1998.

The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise.

### Recommended Books

1. G.B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.
2. G.F. Simmons: Differential Equations with Applications and Historical Notes, McGraw Hill.
3. Sudhir Gorpade and Balmohan Limaye : A course in Multivariable calculus and Analysis, Springer.

# SYLLABUS UNDER AUTONOMY

## MATHEMATICS

SEMESTER 4

Course: A.MAT.4.02

### Linear Algebra

**Learning Objectives:** To understand linear maps and isomorphism between vector spaces, determinant function as an  $n$  form on vector space  $\mathbb{R}^n$  and eigen values and eigen vector of a linear map.

#### Unit 1. Linear transformations.

- (a) Linear transformations- defn and properties and eg including Prjection from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ , rotations and reflections, map defined by matrix, orthogonal projection in  $\mathbb{R}^n$ , functionals.  
The Linear transformation being completely determined by its value on basis.
- (b) Sum and scalar multiple of a linear transformation.  
space  $L(U,V)$  of Linear transformation from  $U$  to  $V$ .  
The dual space  $V^*$ .
- (c) Kernel and image of a linear transformation.  
Rank nullity thm.  
linear isomorphisms and its inverse, composite of a linear transformation.
- (d) Representation of Linear transformation by a matrix wrt an ordered bases.  
Relation between matrices of Linear transformation wrt different ordered bases.  
Matrix of sum, scalar multiple, composite and inverse of Linear transformations.
- (e) Equivalence of rank of a matrix a the linear map associated.  
The dimension of solution space of system of linear equations  $AX = 0$  equals  $n - \text{rank } A$ .

#### Unit 2. Determinants.

- (a) Defn of determinant as an  $n$  linear skew symmetric function and more egs, determinant of a matrix as determinant of its column vectors or row vectors.
- (b) Computation of determinant of  $n \times n$  matrices, properties such as  $\det A^t = \det A$ ,  $\det AB = \det A \det B$ ,  
Laplace expansion of a determinant , vandermonde determinant, determinant of upper and lower triangular matrices.
- (c) Linear dependence and independence of vectors in  $\mathbb{R}^n$  using determinants, Existence and uniqueness of solution of system  $AX = B$  if  $\det A$  is not 0.  
Basic results as  $A \cdot \text{adj}(A) = \det A \cdot I$ .  
Crammers rule.  
Determinant as area and volume.

### Unit 3. Eigen values and eigen vectors.

- (a) Eigen values and eigen vectors of linear map , Eigen values and eigen vectors of  $n \times n$  real matrices.  
Eigen spaces, linear independence of eigen vectors corresponding to distinct eigen values.
- (b) The characteristic polynomial of an  $n \times n$  matrix, characteristic roots.  
Similar matrices, characteristic polynomial of similar matrices. characteristic polynomial of linear map.

References: 1. Introduction to linear algebra by Serge Lang, Springer verlag. 2. Linear algebra a geometric approach by Kumaresan, Prentice-hall of India private limited, New Delhi. 3. Linear algebra by Hoffman and Kunze, Tata McGraw-Hill, New Delhi.