

St. Xavier's College – Autonomous Mumbai

T.Y.B.Sc

Syllabus For 6th Semester Courses in STATISTICS (June 2013 onwards)

Contents:

Theory Syllabus for Courses:

S.STA.6.01 – Probability Distributions and Stochastic Processes.
S.STA.6.02 – Statistical Inference.
S.STA.6.03 – Applied Statistics (I B)
S.STA.6.04 - Applied Statistics (II B)

Practical Course Syllabus for: S.STA.6. PR

SEMESTER 6

COURSE : S.STA.6.01

[60 LECTURES]

PROBABILITY DISTRIBUTIONS AND STOCHASTIC PROCESSES

LEARNING OBJECTIVES :

- 1) To strengthen their concepts in mathematical statistics.
- 2) To prepare students to develop stochastic and queueing models.

UNIT 1 <u>BIVARIATE DISTRIBUTIONS</u> :

- i) Definition and properties of Moment Generating Function of two random variables of continuous and discrete type. Necessary and sufficient condition for independence of two random variables.
- ii) Trinomial distribution:

Definition of joint probability distribution (X,Y). Joint moment generating function, moments μ_{rs} where r = 0,1,2 and s = 0,1,2

Marginal & conditional distributions. Their Means & Variances. Correlation coefficient between the random variables. Distribution of the Sum X+Y.

iii) Multinomial distribution:

Definition of joint probability distribution with parameters $(n, p_1, p_2, ..., p_{k-1})$ where $p_1 + p_2, +..., p_{k-1} + p_k = 1$. Other properties (Concept only)

UNIT 2 <u>BIVARIATE NORMAL DISTRIBUTIONS</u> :

Definition of joint probability distribution (X,Y). Joint moment generating function, moments μ_{rs} where r = 0,1,2 and s = 0,1,2Marginal & conditional distributions. Their Means & Variances. Correlation coefficient between the random variables. Condition for the independence of X and Y. Distribution of aX+bY, where a and b are constants.

SIGNIFICANCE OF CORRELATION COEFFICIENT :

Distribution of sample correlation coefficient when $\rho = 0$. Testing the significance of a correlation coefficient. Fisher's z – transformation. tests for i) $H_0: \rho = \rho_0$ and ii) $H_0: \rho_1 = \rho_2$. Confidence interval for ρ .

UNIT 3 <u>STOCHASTIC PROCESSES</u> :

Definition of stochastic process. Postulates and difference differential equations for the i) Poisson process ii) Pure birth process iii) Yule's process iv) Pure death process v) Poisson type of death process vi) Yule's type / Linear Markovian death process vii) Birth and death process viii) Linear growth model. Derivation of Pn(t), mean and variance wherever applicable.

UNIT 4 QUEUEING THEORY

Basic elements of the Queueing model. Roles of the Poisson and Exponential distributions. Derivation of Steady state probabilities for the birth and death process. Steady state probabilities and the various average characteristics for the following models i) $(M/M/1) : (GD/\infty / \infty)$ ii) $(M/M/1) : (GD/N / \infty)$ iii) $(M/M/c) : (GD/\infty / \infty)$ iv) $(M/M/c) : (GD/N / \infty)$ v) (M/M/c) : (GD/N / N) c < N vi) $(M/M/\infty) : (GD/\infty / \infty)$ Derivation of the waiting time distribution for the $(M/M/1): (FCFS/\infty)$ model

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TOPICS FOR PRACTICALS:

Ser. No.	Topic
1.	Trinomial & Multinomial distributions
2.	Bivariate Normal distribution
3.	Significance of correlation coefficient
4.	Stochastic processes
5.	Queueing theory

<u>REFERENCES</u>:

- 1. Feller W: An introduction to probability theory and it's applications, Volume:1, Third edition, Wiley Eastern Limited.
- 2. Robert V. Hogg & Allen T. Craig: Introduction to Mathematical Statistics, Fifth edition, Pearson Education Pvt Ltd.
- 3. Alexander M Mood, Franklin A Graybill, Duane C. Boes: Introduction to the theory of statistics, Third edition , Mcgraw- Hill Series .
- 4. Hogg R. V. and Tanis E.A. Probability and Statistical Inference Fourth edition McMillan Publishing Company
- 5. S C Gupta & V K Kapoor: Fundamentals of Mathematical statistics, eleventh edition, Sultan Chand & Sons.
- 6. Taha H.A. Operations Research Mcmillan Publishing Co.

7. Kantiswaroop , P.K Gupta and Manmohan, Fourth edition, Sultan Chand & Sons.

- 8. Vohra N.D. Quantitative Techniques in Management Third edition McGraw Hill Companies
- 9. J Medhi: Stochastic Processes, Second edition, Wiley Eastern Ltd.
- 10. Biswas S. Topics in Statistical Methodology Wiley Eastern Ltd.
- 11. J. N. Kapur, H. C. Saxena Mathematical Statistics Fifteenth edition S. Chand and Company

SEMESTER 6

COURSE : S.STA.6.02

STATISTICAL INFERENCE

LEARNING OBJECTIVE : To empower students to validate assumptions made on population parameters.

Unit 1 <u>TESTING OF HYPOTHESIS:</u>

Statistical hypothesis. Problem of testing of hypothesis. Definitions and illustrations of i) Simple hypothesis ii) Composite hypothesis iii) Null Hypothesis iv) (15L) Alternative Hypothesis v)Test of hypothesis vi) Critical region vii) Type I and Type II errors viii) Level of significance ix) p-value x) size of the test xi) Power of the test xii) Power function of a test

xiii) Power curve.

Definition of most powerful test of size α for a simple hypothesis against a simple alternative hypothesis. Neyman –Pearson fundamental lemma. Definition of uniformly most powerful (UMP) test. Construction of UMP test for one tailed alternative hypothesis.

Unit 2 <u>LIKELIHOOD RATIO TEST:</u>

Likelihood ratio principle. Definition of the test statistic and its asymptotic distribution (statement only) Derivation of the test procedure for testing a composite hypothesis against a composite alternative hypothesis for the parameters of Binomial, Poisson, Discrete & Continuous Uniform and Normal distribution.

Unit 3 <u>NON PARAMETRIC TESTS:</u>

Need for non parametric tests. Distinction between a parametric and a non parametric test. Concept of a distribution free statistic. Confidence interval for a quantile. Single sample non parametric test: (i) Sign test (ii) Wilcoxin Signed Rank test. (iii) Run test (iv) Kolmogrov Smirnov test.

Double sample non parametric tests: (i) Sign test (ii) Wilcoxon's signed rank test (iii) Median test (iv) Run test (v) Mann – Whitney –Wilcoxon test.

(vi) Kolmogrov Smirnov test.

Assumptions, justification of the test procedure, critical regions for one tailed and two tailed test procedures.. Problems with no ties.

Unit4 SEQUENTIAL PROBABILITY RATIO TEST:

Sequential probability ratio test procedures for testing a simple null hypothesis against a simple alternative hypothesis. Its comparison with fixed sample size. Most powerful test procedure. Definition of Wald's SPRT of strength (α , β). Problems based on standard distributions such as Bernoulli , Poisson, Normal, Exponential . Graphical and tabular procedures for carrying out the tests. O.C function and A.S.N function and their respective curves.

TOPICS FOR PRACTICALS:

- Testing of Hypotheses.
- Likelihood Ratio Tests.
- Non Parametric Tests.
- Sequential Probability Ratio Test.

[60 LECTURES]

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<u>REFERENCES</u>:

- 1. Hogg R.V. and Craig A.T: Introduction to Mathematical Statistics Fourth edition London Macmillan Co. Ltd.
- 2. Hogg R.V. and Tanis E.A.: Probability and Statistical Inference. Third edition Delhi Pearson Education.
- 3. Daniel W.W. : Applied Non Parametric Statistics First edition Boston-Houghton Mifflin Company
- 4. Sidney Siegal, N. John Castelian Jr. Nonparametric Statistics For Behavioral Sciences, Second edition McGraw Hill International editions
- 5. Wald A.: Sequential Analysis First edition New York John Wiley & Sons
- 6. Biswas S.: Topics in Statistical Methodology. First edition New Delhi Wiley eastern Ltd.
- 7. Gupta S.C. and Kapoor V.K.: Fundamentals of Mathematical Statistics Tenth edition New Delhi S. Chand & Company Ltd.

SEMESTER 6

COURSE: S.STA.6.03

[60 LECTURES]

APPLIED STATISTICS (IB)

LEARNING OBJECTIVE : To orient students on various applications of Statistics in industry.

INVENTORY CONTROL : Unit 1 15LDeterministic Models: Single item static EOQ models for i) Constant rate of demand with instantaneous replenishment, with and without shortages. ii) Constant rate of demand with uniform rate of replenishment, with and without shortages. iii) Constant rate of demand with instantaneous replenishment without shortages, with at most two price breaks. 15L

INVENTORY CONTROL: Unit 2

Probabilistic models. :

Single period with

- i) Instantaneous demand (discrete and continuous) without setup cost.
- ii) Uniform demand (discrete and continuous) without set up cost.

REPLACEMENT THEORY: Unit 3

Replacement of items that deteriorate with time and the value of money:

- i) remains constant
- ii) changes with time (weighted average of costs method). Replacement of items that fail completely.

Individual replacement and Group replacement policies.

GAME THEORY : Unit 4

Definitions of Two person Zero Sum Game, Saddle Point, Value of the Game, Pure and Mixed strategy.

Optimal solution of two person zero sum games: Dominance property,

Derivation of formulae for (2 x 2) game. Graphical solution of (2 x n) and (m x 2) games.

SIMULATION:

Scope of simulation applications. Types of simulation. Monte Carlo Technique of Simulation. Elements of discrete event simulation. Generation of random numbers. Sampling from probability distribution. Inverse method. Generation of random observations from i) Uniform distribution ii) Exponential distribution iii) Gamma distribution

iv) Normal distribution. Simulation techniques applied to inventory and Queueing models.

TOPICS FOR PRACTICALS:

- Deterministic inventory models.
- Probabilistic inventory models.
- Replacement Theory.
- Game Theory.
- Simulation.

<u>REFERENCES</u>:

1. Sharma J. K.: Operations Research Theory and Application, Third edition, Macmillan India Ltd.

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- 2. Sharma S.D. : Operations Research. Eleventh edition, Kedarnath, Ramnath & Co.
- 3. Kantiswaroop, P.K Gupta and Manmohan, Fourth edition, Sultan Chand & Sons.
- 4. V.K. Kapoor. Operations Research.-Techniques for Management. Seventh edition, Sultan Chand & Sons Educational Publishers New Delhi.
- 5. Taha H.A. Operations Research, Sixth edition, Prentice Hall of India Pvt Ltd.
- Vohra N.D. Quantitative Techniques in Management Third edition McGraw Hill 6. Companies
- 7. Bannerjee B. Operation Research Techniques, Second edition, Mumbai Business Books.
- Bronson R. Operations Research , Shaum's Outline series
 Smith P.J. Analysis of Failure and Survival Data.

SEMESTER 6

COURSE: S.STA.6.04

[60 LECTURES]

APPLIED STATISTICS (II B)

LEARNING OBJECTIVE : To enable students to develop the technique of model building

Unit 1	 MULTIPLE LINEAR REGRESSION – 1: Multiple linear regression model with two independent variables: Assumptions of the model, Derivation of ordinary least square (OLS) estimators of regression coefficients. Properties of least square estimators (without proof) Concept of multiple correlation, partial correlation, R² and adjusted R². Properties of multiple and partial correlation coefficients. Testing the significance of multiple and partial correlation coefficients. Procedure of testing overall significance of the model significance of individual coefficients significance of contribution of additional independent variable to a model. Confidence intervals for the regression coefficients 	15L
Unit 2	MULTIPLE LINEAR REGRESSION – II : <u>Autocorrelation:</u> Concept, Detection using i) Run Test ii) Durbin Watson Test, Consequences of using OLS estimators in presence of autocorrelation, Generalized least square(GLS) method. <u>Heteroscedasticity:</u> Concept,Detection using i) Spearman's rank correlation test ii) Breusch – Pagan – Godfrey Test. Consequences of using OLS estimators in presence of heteroscedasticity Weighted least square (WLS) estimators <u>Multicollinearity:</u> Concept, Detection using R square & t ratios, simple correlation coefficients, Variance Inflation Factor (VIF) Consequences of using OLS estimators in presence of multi collinearity.	15L
Unit 3	TIME SERIES : Definition of Time series. Its components. Models of Time Series. Estimation of trend by i) Freehand curve method ii) Method of semi averages iii) Method of moving averages iv) Method of least squares. v) Exponential smoothing method Estimation of seasonal component by i) Method of simple averages ii) Ratio to moving average method iii) Ratio to trend method	15L
Unit 4	 RELIABILITY: Concept of reliability or survival function, Hazard function, Cumulative hazard function Life time distributions :i) Exponential ii) Gamma iii) Weibull iv) Gumbel. Definitions of increasing (decreasing) failure rate. Observations schemes and censoring: left and right censoring, interval censoring, Type I, Type II, random right censoring. Concept of truncation. Kaplan-Meier estimator of survival function and median survival time. Plotting of K-M estimator. Kolmogrov-Smirnov goodness of fit test. Reliability: Structure function, coherent system, standard systems: series, parallel, k-out-of-n system of independent components having exponential life distributions. Mean Time to Failure of a system (MTTF). 	15L

TOPICS FOR PRACTICALS:

- Multiple regression model.
- Autocorrelation, Heteroscedasticity, Multicollinearity.
- Time series..
- Reliability.

<u>REFERENCES</u>:

- 1. S.C.G upta , V.K.Kapoor : Fundamentals of Applied Statistics, Third edition, Sultan Chand & Sons.
- 2. Barlow R.E. and Prochan Frank.: Statistical Theory of Reliability and Life Testing, First edition, John Wiley & Sons
- 3. Mann N.R., Schafer R.E., Singapurwalla N.D. : Methods for Statistical Analysis of Reliability and Life Data., First edition, Wiley International
- 4. Damodar Gujrathi : Basic Econometrics, Second edition McGraw-Hill Companies.
- 5. S.M.Ross: Probability Models & Applications.
- 6. A.M.Goon, M.K.Gupta, B.Dasgupta: Fundamentals of Statistics, Vol Two, Fifth Revised edition, The World Press Pvt Ltd.
- 7. Smith P.J: Analysis of Failure and Survival Data
- 8. Daniel W.W : Applied Non Parametric Statistics First edition Boston-Houghton Mifflin Company