St. Xavier's College – Autonomous Mumbai

Syllabus
For Ist Semester Courses in MATHEMATICS

(June 2016 onwards)

Contents:

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.1.01**

Title: CALCULUS – I

Learning Objectives: To learn about (i) lub axiom of R and its consequences.
(ii) Convergence of sequences in R.
(iii) Limit and continuity of real valued functions of one variable.

Number of lectures : 45

Unit-I: Real Number System and Sequence of Real Numbers **(15 Lectures)**

Real Numbers: Real number system and order properties of R, Absolute value properties AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhood, Hausdorff property, Bounded sets, Continuum property (l.u.b.axiom–statement, g.l.b.) and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, Density theorem.

Sequences: Definition of a sequence and examples, Convergence of a sequence, every convergent sequence is bounded, Limit of a sequence, uniqueness of limit if it exists, Divergent sequences, Convergence of standard sequences like $\{n^{1/n}\}$, $\{a^n\}$, Sequential version of Bolzano-Weierstrass theorem.

Unit II: Sequences (contd.) **(15 Lectures)**

Algebra of convergent sequences, Sandwich theorem for sequences, Monotone sequences, Monotone Convergence theorem and its consequences such as convergence of $\left(1+\frac{1}{n}\right)^n$.

Subsequences: Definition, Subsequence of a convergent sequence is convergent and converges to the same limit.

Cauchy sequence: Definition, every convergent sequence is a Cauchy sequence and converse.

Unit III: Limits and Continuity of real valued functions of one variable **(15 Lectures)**

Limit of Functions: Graphs of some standard functions such as $|x|$, e^x, log x, $\frac{1}{x}$, ax^2+bx+c, x^3, $\lfloor x\rfloor$ (Flooring function), $\lceil x\rceil$ (Ceiling function), sin x, cos x, tan x, x sin(1/x), x^2 sin(1/x) over suitable intervals, Graph of a bijective function and its inverse, Limit of a function, evaluation of limit of simple functions using $\in -\delta$ definition, uniqueness of limit if it exists, Algebra of limits (with proof), Limit of a composite functions, Sandwich theorem (only statement), Left hand and right hand limits, non–existence of limits, Limit as $x \to \pm\infty$.

Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, $\in -\delta$ definition of continuity, Sequential continuity, Algebra of continuous functions, Continuity of composite functions. Discontinuous functions, examples of removable and essential discontinuity.

Recommended Books:

1. R. R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. Robert G. Bartle and Donald R. Sherbet : Introduction to Real Analysis, Springer Verlag.
4. Howard Anton, Calculus - A new Horizon, Sixth Edition, John Wiley and Sons Inc,1999.

Additional Reference Books

1. T. M. Apostol, Calculus Vol I, Wiley & Sons (Asia) Pte. Ltd.
2. Courant and John, A Introduction to Calculus and Analysis, Springer.
3. Ajit and Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/cole Publishing Company,1994.
5. Sudhir R. Ghorpade and Balmohan V. Limaye, A Course in Calculus and Real Analysis, Springer International Ltd, 2000.

Assignments (Tutorials)

1. Application based examples of Archimedean property, intervals, neighbourhood.
2. Consequences of continuum property, infimum and supremum of sets.
3. Calculating limits of sequence.
4. Cauchy sequence, monotone sequence.
5. Limit of a function and Sandwich theorem.
6. Continuous and discontinuous functions.

F.Y.B.Sc. – Mathematics **Course Code: S.MAT.1.02**

Title: ALGEBRA

Learning Objectives: To learn about (i) divisibility of integers.
(ii) properties of equivalence relations and partitions.
(iii) roots of polynomials.

Number of lectures : 45

Prerequisites:

Set Theory: Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations and combinations.

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I: Integers and divisibility **(15 Lectures)**

Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle. Divisibility in integers, division algorithm, greatest common divisor (g.c.d.) and least common multiple (l.c.m.) of two integers, basic properties of g.c.d. such as existence and uniqueness of g.c.d. of integers a and b and that the g.c.d. can be expressed as ma + nb where m, n are in Z, Euclidean algorithm, Primes, Euclid's lemma, Fundamental theorem of arithmetic, the set of primes is infinite. Congruences, definition and elementary properties, Euler's ϕ function, Statements of Euler's theorem, Fermat's theorem and Wilson theorem, Applications.

Unit II: Functions and Equivalence relations **(15 Lectures)**

Definition of a function, domain, codomain and range of a function, composite functions, examples, Direct image f [A] and inverse image $f^{-1}[A]$ of a function. Injective, surjective, bijective functions, Composite of injective, surjective, bijective functions, Invertible functions, Bijective functions are invertible and conversely, Examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples. Equivalence relations, Equivalence classes, properties such as two equivalences classes are either identical or disjoint. Definition of partition, every partition gives an equivalence relation and vice versa, Congruence an equivalence relation on Z, Residue classes, Partition of Z, Addition modulo n, Multiplication modulo n, examples, conjugate classes.

Unit III: Polynomials (15 Lectures)

Definition of polynomial, polynomials over F where F = Q , R ,C. Algebra of polynomials, degree of polynomial, basic properties, Division algorithm in F[X](without proof) and g.c.d. of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem, A polynomial of degree n over F has at most n roots. Complex roots of a polynomial in R[X] occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree n in R[X] has exactly n complex roots counted with multiplicity. A non-constant polynomial in R[X] can be expressed as a product of linear and quadratic factors in C[X]. Necessary condition for a rational number to be a root of a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is an irrational number where p is a prime number, n^{th} roots of unity, sum of n^{th} roots of unity.

Recommended Books

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.
2. Norman L. Biggs, Discrete Mathematics, Revised Edition, Clarendon Press, Oxford 1989.

Additional Reference Books

1. I. Niven and S. Zuckerman, Introduction to the theory of numbers, Third Edition, Wiley Eastern, New Delhi, 1972.
2. G. Birkoff and S. Maclane, A Survey of Modern Algebra, Third Edition, Mac Millan, New York, 1965.
3. N. S. Gopalkrishnan, University Algebra, Ne Age International Ltd, Reprint, 2013.
4. I .N. Herstein, Topics in Algebra, John Wiley, 2006.
5. P. B. Bhattacharya S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, New Age International, 1994.
6. Kenneth Rosen, Discrete Mathematics and its applications, Mc-Graw Hill International Edition, Mathematics Series.

Assignments (Tutorials)

1. Mathematical induction (The problems done in F.Y.J.C. may be avoided).
2. Division Algorithm and Euclidean algorithm, in Z, Primes and the Fundamental Theorem of Arithmetic.
3. Functions (direct image and inverse image). Injective, surjective, bijective functions, finding inverses of bijective functions.
4. Congruences and Euler's ϕ function, Fermat's little theorem, Euler's theorem and Wilson's theorem.

5. Equivalence relation.
6. Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

CIA I – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

CIA II – 20 marks, 45 mins. (Objectives/Short questions, not more than 5 marks each)

End Semester exam – 60 marks, 2 hours.

There will be three questions, one per unit. The Choice is internal- i.e. within a unit and could be between 50% to 100%